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Supersymmetry and electroweak breaking in the interval

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Abstract: Hypermultiplets are considered in the five-dimensional interval where all fields are continuous and the boundary conditions are dynamically obtained from the action principle. The orbifold boundary conditions are obtained as particular cases. We can interpret the Scherk-Schwarz supersymmetry breaking as a misalignment of boundary conditions while a new source of supersymmetry breaking corresponding to a mismatch of different boundary parameters is identified. The latter can be viewed as coming from boundary supersymmetry breaking masses for hyperscalars and the nature of the corresponding supersymmetry breaking parameter is analyzed. For some regions of the parameter space where supersymmetry is broken (either by Scherk-Schwarz boundary conditions or by boundary hyperscalar masses) electroweak symmetry breaking can be triggered at the tree level.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Beyond Standard Model.

Contents

1. Introduction

The existence of extra dimensions is a general prediction of fundamental (string) theories that aim to unify all interactions, including gravity, and provide a consistent quantum description of them. If the radii of these extra dimensions is as large as the 1/TeV scale [1], matter can propage in the bulk and the very existence of extra dimensions can provide new mechanisms for supersymmetry and electroweak breaking [2]. In five and six-dimensional theories gauge bosons are located in vector multiplets and matter and Higgs bosons in hypermultiplets. While supersymmetry breaking should be felt primarily by $SU(2)_R$ doublets (fermions in vector multiplets and bosons in hypermultiplets), electroweak breaking by the conventional Higgs mechanims concerns the bosons of the Higgs hypermultiplet. Therefore for a better understanding of supersymmetry and electroweak breaking we should consider mainly the system of Higgs hypermultiplets propagating in the bulk. In this paper we will study propagation of hypermultiplets in five dimensions (5D).

Propagation of matter in the bulk of the fifth dimension has been widely considered in the past [3, 4]. The 5D space-time, with coordinates (x^μ, y), is often constructed as the orbifold $S^1/\mathbb{Z}_2$, where points on the circle of radius R[1] related by the reflection of the fifth coordinate $y \to -y$ are identified. In this approach ("upstairs" approach) fields are classified according to the $\mathbb{Z}_2$ parity and their boundary conditions (BC's) at the fixed points imposed. In the orbifold approach the space is singular at the orbifold fixed points $(y = 0, \pi)$, it has no boundaries and the fields satisfy the circle periodicity and the orbifold parity. In this approach mass terms localized at the fixed points can trigger supersymmetry breaking and make the fermionic wave functions to make discontinuous jumps at them [5, 6].

An alternative approach is working in the fundamental region of the orbifold $[0, \pi]$ and giving up the rigid orbifold BC's [7]. In this approach ("downstairs" approach) fields have no defined parity and BC's are dynamically determined by the action principle [8]. The space is not singular but has boundaries at $y = 0, \pi$ and wave functions are continuous even in the presence of boundary mass terms. In particular the propagation of gauge fermions in the interval has been considered in ref. [8] where the Scherk-Schwarz (SS) supersymmetry breaking [9] was interpreted as misalignment of BC's at the two boundaries that departure from supersymmetry in boundary parameters.

In this paper we will consider propagation of hypermultiplets in the interval. We will construct a globally supersymmetric action and identify possible sources of supersymmetry breaking corresponding to departure from supersymmetry of boundary parameters. New patterns of supersymmetry breaking will arise that can trigger electroweak breaking at the tree-level. While SS-breaking can be understood as a misalignment of boundary conditions at different branes and hence is a nonlocal breaking, the other source of SUSY breaking is purely localized at one (or both) boundaries: it can be traced back to the fact that supersymmetry transformations do not leave this class of boundary conditions invariant. Nevertheless, the bulk plus boundary action is formally supersymmetric and it is tempting to speculate that this breaking is spontaneous. Since a complete thorough understanding of this effect at the supergravity level is beyond the scope of the present paper, we point out that there is a rather simple and completely satisfactory descrition in the upstairs (orbifold) approach: all boundary conditions are fixed by the parity assignments stable under supersymmetry [10], while localized mass terms for hyperscalars can mimic the downstairs boundary conditions through jumps across the brane. Since there is a simple understanding of such soft mass terms as a localized 4D supersymmetry breaking mechanism, we consider this local kind of breaking as a viable source of supersymmetry breaking although a complete supergravity understanding is still lacking.

The structure of the paper is as follows. In section 2 the general supersymmetric formalism for hypermultiplets in the interval will be worked out. The equations of motion for hyperscalars and hyperfermions will be solved in section 3 where mass eigenvalues and eigenfunctions are explicitly obtained. In particular the conditions for supersymmetry will be established while supersymmetry breaking by boundary conditions will be considered in detail in section 4. A comparison with the orbifold approach will be done in section 5.

[1] We will work from here on, unless explicitly stated, in units where $R \equiv 1$.

The nature of supersymmetry breaking by boundary hyperscalar masses is clarified in section 6 where a simple toy model is constructed based on a U(1) gauge theory under which hypermultiplets transform. The problem of embedding $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ in the interval will be considered in section 7 where the interface between supersymmetry and electroweak breaking will be studied, including the tree-level prediction for the Higgs mass. Finally section 8 contains our conclusions and some (technical) useful identities are presented in appendix A.

2. Hypermultiplets in the interval

In this section we will consider the formalism for a single hypermultiplet propagating in the interval. There are two equivalent approaches. One is to consider the fields in the hypermultiplet as complex and unconstrained fields: it is the so-called complex hypermultiplet [11]. The second approach is to introduce an $\mathrm{SU}(2)_H$ index on the hypermultiplet fields[2]

$$\mathbb{H}^\alpha = (\Phi_i, \Psi, F_i)^\alpha \,, \tag{2.1}$$

that transforms as a doublet, and to introduce on the fields the reality constraint [12]

$$\bar{\Phi}^i_\alpha \equiv (\Phi^\alpha_i)^* = \epsilon^{ij}\epsilon_{\alpha\beta}\Phi^\beta_j \,. \tag{2.2}$$

The auxiliary fields obey the same constraint, while the hyperfermions now obey a symplectic Majorana constraint with respect to the new $\mathrm{SU}(2)_H$[3]

$$\bar{\Psi}_\alpha \equiv (\Psi^\alpha)^\dagger\gamma^0 = \epsilon_{\alpha\beta}(\Psi^\beta)^T C \,, \tag{2.3}$$

where C is the 5D charge conjugation matrix. In the following we will use real hypermultiplets and conventions and notations are those of ref. [13]. In principle we always could explicitely solve the reality constraints to obtain the standard complex hypermultiplet, but we will find it useful to express our results in terms of the real fields. It is important to realize that the doublet of real fields describes the same degrees of freedom as one complex hypermultiplet.

We will consider the total action $\mathcal{S} = \mathcal{S}_{\rm bk} + \mathcal{S}_{\rm bd}$ as the sum of a bulk ($\mathcal{S}_{\rm bk} = \mathcal{S}^0_{\rm bk} + \mathcal{S}^{\rm m}_{\rm bk}$) and a boundary ($\mathcal{S}_{\rm bd}$) term, as

$$\mathcal{S}^0_{\rm bk} = \int_{\mathcal{M}} \left(-\frac{1}{2}\bar{\Phi}\,\partial^2\Phi + \frac{i}{2}\bar{\Psi}\gamma^M\partial_M\Psi + 2\bar{F}F \right) \tag{2.4}$$

$$\mathcal{S}^{\rm m}_{\rm bk} = \int_{\mathcal{M}} \left(2i\bar{F}\mathcal{M}\Phi + \frac{1}{2}\bar{\Psi}\mathcal{M}\Psi \right) \tag{2.5}$$

$$\mathcal{S}_{\rm bd} = \int_{\partial\mathcal{M}} \left(\frac{1}{4}\bar{\Psi}S\Psi + \frac{1}{4}(\bar{\Phi}R\Phi)' + \frac{1}{4}\bar{\Phi}N(-1+R)\Phi \right) , \tag{2.6}$$

where $\mathcal{S}^{\rm m}_{\rm bk}$ in (2.5) is a supersymmetric bulk mass action and for simplicity we are using an indexless notation. S and $\mathcal{M}$ are matrices in the $\mathrm{SU}(2)_H$ indices, R has matrix indices

[2] The subscript i transforms as a doublet under the group $\mathrm{SU}(2)_R$.

[3] The convention is such that $\epsilon^{12} = \epsilon_{12} = +1$.

in both $\mathrm{SU}(2)_H$ and $\mathrm{SU}(2)_R$ and N is a real number.[4] S and R are dimensionless matrices and N and $\mathcal{M}$ have dimension of mass. We will show in subsection 2.2 that this action is supersymmetric. In the next section we will first derive the BC's resulting from the action (2.4), (2.5) and (2.6).

2.1 The boundary conditions

We take S, $\mathcal{M}$ and R to be hermitean. In order for the action (2.4), (2.5) and (2.6) to be real, the reality constraints imply that S, $\mathcal{M}$ and R satisfy:

$$S_\alpha^\beta = \epsilon_{\alpha\gamma}\epsilon^{\beta\delta}S_\delta^\gamma \tag{2.7}$$
$$\mathcal{M}_\alpha^\beta = \epsilon_{\alpha\gamma}\epsilon^{\beta\delta}\mathcal{M}_\delta^\gamma \tag{2.8}$$
$$R_{i\alpha}^{j\beta} = \epsilon_{ik}\epsilon^{jl}\epsilon_{\alpha\gamma}\epsilon^{\beta\delta}R_{k\delta}^{l\gamma} \tag{2.9}$$

With this choice, all terms in the above action are real without partial integration because of the reality constraints. We now make the ansatz[5] $R = T \otimes S$ where T acts on $\mathrm{SU}(2)_R$ only, i.e. T^i_j. Then all conditions on R, S and $\mathcal{M}$ can be formulated in matrix notation as

$$\mathcal{M}^\dagger = \mathcal{M}\,, \qquad \mathcal{M}^T = -\sigma^2\mathcal{M}\sigma^2 \tag{2.10}$$
$$S^\dagger = S\,, \qquad S^T = -\sigma^2 S\sigma^2 \tag{2.11}$$
$$T^\dagger = T\,, \qquad T^T = -\sigma^2 T\sigma^2 \tag{2.12}$$

The solution to these constraints are

$$\mathcal{M} = M\,\vec{p}\cdot\vec{\sigma}, \qquad S = \vec{s}\cdot\vec{\sigma}, \qquad T = \vec{t}\cdot\vec{\sigma}\,, \tag{2.13}$$

where $\vec{s}$, $\vec{p}$ and $\vec{t}$ are real dimensionless vectors, M is a mass parameter and $\vec{p}$ a unit vector. All calculations can now be performed without writing explicit indices by use of the identities presented in the Appendix.

Variation of the action gives a bulk and a boundary term $\delta\mathcal{S} = \delta_{\rm bk}\mathcal{S} + \delta_{\rm bd}\mathcal{S}$ the latter coming from the partial integration of the variation of the bulk action and from the variation of the boundary action:

$$\delta_{\rm bd}\mathcal{S} = \frac{1}{2}\int_{\partial\mathcal{M}}\left[\delta\bar{\Psi}(i\gamma^5 + S)\Psi + \delta\bar{\Phi}(-1+R)\Phi' + \delta\bar{\Phi}'(1+R)\Phi + \delta\bar{\Phi}\,N(-1+R)\Phi\right]. \tag{2.14}$$

The BC's resulting from this are

$$\left(1 + i\gamma^5 S\right)\Psi = 0\,, \qquad \bar{\Psi}\left(1 - i\gamma^5 S\right) = 0 \tag{2.15}$$
$$(1+R)\,\Phi = 0\,, \qquad \bar{\Phi}\,(1+R) = 0 \tag{2.16}$$
$$(-1+R)\left[\Phi' + N\Phi\right] = 0\,, \qquad \left[\bar{\Phi}' + N\bar{\Phi}\right](-1+R) = 0\,. \tag{2.17}$$

[4]It is understood that S, R and N can take on different values at the two branes, i.e. the usual index $f = 0, \pi$ is suppressed here.

[5]The choice $R = T \otimes S$ is motivated by the fact that the supersymmetry transformation laws (see subsection 2.2) make sense. The boundary conditions we will find are the same on both sides of the transformation laws provided ϵ fulfills $(1 + i\gamma^5 T)\epsilon = 0$.

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We find that for consistent fermionic BC's $\vec{s}$ has to be a unit vector [8]. Furthermore, eqs. (2.16)–(2.17) give rise to eight real BC's at each brane, whereas we need only four. Thus the bosonic system is clearly overdetermined unless the 8×8 matrix

$$\begin{pmatrix} 0 & 1+R \\ -1+R & N(-1+R) \end{pmatrix}. \tag{2.18}$$

is singular. Its determinant is given by $(1-\vec{s}^2\vec{t}^2)^4$, which vanishes if $\vec{t}$ is a unit vector. In fact, it is easy to see that in this case $(1+R)/2$ and $(1-R)/2$ form mutually orthogonal projectors on two-dimensional subspaces, and hence we reduce the number of independent BC's on each brane down to four.

Standard orbifold BC's are obtained as particular cases by taking $S_f = s_f\,\sigma_3$, $T_f = t_f\,\sigma_3$, $N = 0$. The (independent) parity eigenstates are then given by $\varphi = \Phi^1_1$, $\varphi^c = \Phi^2_1$, $\psi_L = \Psi^1_L$, $\psi_R = \Psi^1_R$ and their parities are

$$\begin{aligned} \varphi(y_f+y) &= -\,s_f\,t_f\,\varphi(y_f-y)\,, \\ \varphi^c(y_f+y) &= +\,s_f\,t_f\,\varphi^c(y_f-y)\,, \\ \psi_L(y_f+y) &= -\,s_f\,\psi_L(y_f-y)\,, \\ \psi_R(y_f+y) &= +\,s_f\,\psi_R(y_f-y)\,. \end{aligned}$$

However, our formalism allows for more general BC's. As we will see, it can produce SS-twists in both $\mathrm{SU}(2)_R$ as well as $\mathrm{SU}(2)_H$ space with SS parameter given by the angle between the vectors $\vec{t}_0$, $\vec{t}_\pi$ and $\vec{s}_0$, $\vec{s}_\pi$ respectively. Furthermore, we can have mixed BC's for bosons parametrized by the masses N_f. A more detailed and general comparison with the orbifold approach will be done in section 5.

2.2 Supersymmetry of the action

We now want to show that the action (2.4)–(2.6) is indeed supersymmetric. The transformation laws are given by

$$\begin{aligned} \delta\Phi^\alpha_i &= i\bar{\epsilon}_i\Psi^\alpha \\ \delta\Psi^\alpha &= -\gamma^M\epsilon^i\partial_M\Phi^\alpha_i + 2\epsilon^i F^\alpha_i \\ \delta F^\alpha_i &= -\frac{i}{2}\bar{\epsilon}_i\gamma^M\partial_M\Psi^\alpha\,. \end{aligned} \tag{2.19}$$

First consider the bulk part, eqs. (2.4)–(2.5). Under supersymmetry the lagrangian varies into a total derivative which leaves a brane variation given by

$$\delta_\epsilon\mathcal{S}_{\rm bk} = \int_{\partial\mathcal{M}}\left(-\bar{F}\bar{\epsilon}(i\gamma^5)\Psi + \frac{1}{2}\bar{\Phi}\bar{\epsilon}\gamma^\mu\partial_\mu(i\gamma^5)\Psi + \frac{i}{2}\bar{\Phi}\bar{\epsilon}\Psi' - \bar{\epsilon}\bar{\Phi}\mathcal{M}\gamma^5\Psi\right). \tag{2.20}$$

Now consider the variation of the boundary action, eq. (2.6).

$$\begin{aligned} \delta_\epsilon\mathcal{S}_{\rm bd} = \int_{\partial\mathcal{M}}\Big(&-\frac{1}{2}\bar{\epsilon}\gamma^\mu\partial_\mu\bar{\Phi}S\Psi - \frac{1}{2}\bar{\epsilon}\gamma^5\bar{\Phi}'S\Psi + \\ &+\bar{\epsilon}\bar{F}S\Psi + \frac{i}{2}\bar{\Phi}'R\bar{\epsilon}\Psi + \frac{i}{2}\bar{\Phi}R\bar{\epsilon}\Psi' + \frac{i}{2}N\bar{\Phi}(-1+R)\bar{\epsilon}\Psi\Big)\,. \end{aligned} \tag{2.21}$$

Discarding a total 4D derivative we can rewrite the sum of (2.20) and (2.21) as

$$\delta_\epsilon \mathcal{S} = \int_{\partial\mathcal{M}} \left(\frac{i}{2}\bar{\epsilon}\bar{\Phi}(1+R)\Psi' + \frac{i}{2}(\bar{\Phi}' + N\bar{\Phi})(-1+R)\bar{\epsilon}\Psi - \right.$$
$$\left. - \frac{1}{2}\bar{\epsilon}\gamma^5\bar{\Phi}\mathcal{M}(1+i\gamma^5 S)\Psi - i\bar{\epsilon}\gamma^5\left(\bar{F} - \frac{i}{2}\bar{\Phi}\mathcal{M}\right)(1-i\gamma^5 S)\Psi \right). \quad (2.22)$$

Using the BC's (2.15), (2.16) and (2.17) the first three terms vanish. Finally we use the EOM for F

$$F = -\frac{i}{2}\mathcal{M}\Phi, \qquad \bar{F} = \frac{i}{2}\bar{\Phi}\mathcal{M} \quad (2.23)$$

to deduce that the whole variation is zero.

3. Equations of motion and the spectrum

In this section we will solve the equations of motion (EOM) in the bulk for the bosonic and fermionic sectors of the hypermultiplet and impose on the solutions the corresponding BC's on both boundaries. We will obtain as a result the mass eigenfunctions and eigenvalues for the different modes.

3.1 Hyperscalars

We first make a general mode decomposition of bosonic fields as

$$\Phi_i^\alpha(x,y) = \sum_n f_{i,n}^\alpha(y)\, \phi_n(x)\,, \quad (3.1)$$

where $\phi_n(x)$ is the real 4D mass eigenstate corresponding to the mass m_n.[6]. The solution to the EOM arising from the bulk action (2.4) and (2.5) is given by

$$f(y) = \cos(\Omega y)\, a + \sin(\Omega y)\, b\,, \quad (3.2)$$

where a_i^α and b_i^α are constant matrices and $\Omega_n = \sqrt{m_n^2 - M^2}$.

The BC's at $y = 0$, eqs. (2.16) and (2.17) yield

$$\begin{aligned} a &= (1 - R_0)\,\varphi \\ b &= \left[1 + R_0 - \frac{N_0}{\Omega}(1 - R_0)\right]\varphi\,, \end{aligned} \quad (3.3)$$

where φ is a constant unconstrained matrix. Imposing the BC's at $y = \pi$ determines φ and gives the discrete eigenvalue spectrum as a function of $\vec{s}_f$ and $\vec{t}_f$. Defining the angles ω and $\tilde{\omega}$ by

$$\begin{aligned} \vec{s}_0 \cdot \vec{s}_\pi &= \cos(2\pi\tilde{\omega})\,, \\ \vec{t}_0 \cdot \vec{t}_\pi &= \cos(2\pi\omega)\,, \end{aligned} \quad (3.4)$$

[6] From here on and for notational simplicity we will drop, unless explicitly stated, the subscript n as well as we will use the compact notation where the i and α indices are omitted.

the bosonic mass eigenvalues are given as the solutions of the equation

$$A(\omega+\tilde{\omega},N_0,N_\pi)A(\omega-\tilde{\omega},N_0,N_\pi)=0$$
$$A(\varphi,N_0,N_\pi)=\sin^2(\pi\varphi)\;-\;\frac{N_0-N_\pi}{\Omega}\,\tan(\Omega\pi)-\left[\cos^2(\pi\varphi)+\frac{N_0\,N_\pi}{\Omega^2}\right]\tan^2(\Omega\pi)\,. \quad (3.5)$$

Notice that the parameter ω describes a Scherk-Schwarz twist and thus it corresponds to supersymmetry breaking. The parameter $\tilde{\omega}$ is a twist in the global $\mathrm{SU}(2)_H$ symmetry and amounts to a supersymmetric mass, as we will see in the next section. As for the mass parameters N_f they can conserve or break supersymmetry depending on their relation with $M\,\vec{p}\cdot\vec{s}_f$ as we will see. For special values of $(\omega,\tilde{\omega})$ the mass formula becomes a perfect square, indicating a degeneracy in the spectrum. The values where this happens are given by $(\omega,0)$, $(\omega,\frac{1}{2})$, $(0,\tilde{\omega})$ and $(\frac{1}{2},\tilde{\omega})$.

We want to close this section by noticing that the condition for the existence of an exactly massless mode is given by the equation

$$(n_0-\tau^{-1})(n_\pi+\tau^{-1})=\cos^2\pi(\omega\pm\tilde{\omega})(1-\tau^{-2})\,, \quad (3.6)$$

where

$$n_f=\frac{N_f}{M} \quad (3.7)$$

and $\tau\equiv\tanh(M\pi)$. This defines a hyperbola which divides the (n_0,n_π) plane in regions where the lightest mode is tachyonic or physical respectively. This will be discussed in more detail in section 4.

3.2 Hyperfermions

We will define the Dirac fermion in the 5D action as

$$\Psi^\alpha=\begin{pmatrix}\chi^\alpha\\ \bar{\psi}^\alpha\end{pmatrix} \quad (3.8)$$

where χ^α and $\bar{\psi}^\alpha$ are Weyl fermions subject to the reality condition $\psi_\alpha=\epsilon_{\alpha\beta}\chi^\beta$. We will make the mode decomposition

$$\chi^\alpha(x,y)\;=\;\sum_n f_n^\alpha(y)\chi_n(x)$$
$$\bar{\psi}^\alpha(x,y)\;=\;\sum_n g_n^\alpha(y)\bar{\chi}_n(x)\,, \quad (3.9)$$

where $(\chi_n(x),\bar{\chi}_n(x))^T$ is the 4D Majorana spinor with mass eigenvalue m_n. We now define the vector

$$h(y)=\begin{pmatrix}f(y)\\ g(y)\end{pmatrix} \quad (3.10)$$

where we have dropped the indices α, n. The bulk EOM corresponding to the bulk action (2.4) and (2.5) has the solution

$$h(y)=\mathcal{U}(y)\;h(0);\quad \mathcal{U}(y)=\cos(\Omega y)+(im\sigma^2+\mathcal{M}\sigma^3)\,\frac{\sin(\Omega y)}{\Omega}\,, \quad (3.11)$$

where $\sigma^{2,3}$ are acting on the space of eq. (3.10) and $\mathcal{M}$ is acting on $\mathrm{SU}(2)_H$ indices.

We now apply the BC's (2.15) at the two boundaries. In particular the BC's at $y = 0$ imply that

$$h(0) = \left(1 - \sigma^3 S_0\right)\, \tilde{h} \tag{3.12}$$

and at $y = \pi$

$$V(\pi)\, \tilde{h} \equiv \left(1 + \sigma^3 S_\pi\right) \mathcal{U}(\pi) \left(1 - \sigma^3 S_0\right)\, \tilde{h} = 0 \tag{3.13}$$

The 4×4 matrix $V(\pi)$ has rank $r \leq 2$ because it is proportional to the projector $(1 + \sigma^3 S_\pi)/2$. The existence of a non-trivial solution requires $r = 1$ which provides the constraint satisfied by the mass eigenvalues. The result can be expressed in terms of the angles $\tilde{\omega}$ defined in eq. (3.4) and α_f defined by

$$\vec{p} \cdot \vec{s}_f = \cos(2\pi\alpha_f) \equiv c_f\,. \tag{3.14}$$

The fermion mass eigenvalues satisfy then the equation

$$1 - \tilde{c} - 2\,(c_0 - c_\pi)\frac{M}{\Omega}\tan(\Omega\pi) - \left[1 + \tilde{c} + 2\,c_0\,c_\pi\frac{M^2}{\Omega^2}\right]\tan^2(\Omega\pi) = 0\,, \tag{3.15}$$

where the quantity $\tilde{c} = \cos(2\pi\tilde{\omega})$. Note that the quantities c_0 and c_π are not completely independent but are bound to lie inside an elliptical disk

$$\frac{(c_0 + c_\pi)^2}{\cos^2\pi\tilde{\omega}} + \frac{(c_0 - c_\pi)^2}{\sin^2\pi\tilde{\omega}} \leq 4\,. \tag{3.16}$$

This condition stems from the fact that the three angles between the three vectors $\vec{s}_f$, $\vec{p}$ are not independent but rather constrained by triangle inequalities. For instance, if $\tilde{c} = 1$ it is clear that $0 \leq c_0 = c_\pi \leq 1$ (in this case the ellipse actually shrinks to a line). It will also be convenient to express the condition eq. (3.15) in terms of the function A defined in eq. (3.5):

$$A(\tilde{\omega}, c_0 M, c_\pi M) = 0\,. \tag{3.17}$$

The bosonic, eq. (3.5), and fermionic, eq. (3.17), spectra can easily encompass the cases already studied in the literature. For instance for the particularly simple case where $N_f = M = 0$ the bosonic spectrum provided by eq. (3.5) is given by $m_n = n \pm \omega \pm \tilde{\omega}$ while the fermionic one, provided by eq. (3.15), is given by $m_n = n \pm \tilde{\omega}$ in agreement with the results of the model studied in ref. [3].

For the case $\tilde{\omega} = 0$, $c_0 = c_\pi = 1$ (i.e. the three vectors $\vec{s}_f$, $\vec{p}$ aligned) and $N_f = M$ the bosonic spectrum from eq. (3.5) is given by the solution of

$$\sin^2(\pi\omega) = \frac{\Omega^2 + M^2}{\Omega^2}\sin^2(\Omega\pi) \tag{3.18}$$

while the fermionic spectrum is given by $m_n^2 = n^2 + M^2(1 - \delta_{n0})$ in agreement with the results in ref. [14]. While other cases can be easily studied using the general equations we will next concentrate in particularly interesting cases for physics purposes. In particular, we will examine how supersymmetry can be broken and how vectorlike fermions can arise.

4. Supersymmetry breaking

The bosonic spectrum described as the solution of eq. (3.5) depends on four dimensionless parameters: ω, $\tilde{\omega}$ and n_f. Similarly the fermionic spectrum described as the solution of eq. (3.15) depends on three parameters: $\tilde{\omega}$ and α_f. The parameter ω is a genuine Scherk-Schwarz supersymmetry breaking parameter while a particular relation between n_f and α_f can conserve/break supersymmetry as we will see in this section. Finally $\tilde{\omega}$ affects to both bosons and fermions and can play the role of a supersymmetric mass if the only source of supersymmetry breaking is the parameter ω as we have seen in the simple example described at the end of the previous section.

Comparison between (3.5) and (3.17) dictates the supersymmetric relation between n_f and $c_f = \cos(2\pi\alpha_f)$. Indeed this is given by

$$n_f = c_f \,. \tag{4.1}$$

If n_f does not satisfy the relations (4.1) supersymmetry is broken. In fact even if the lagrangian is (on-shell) supersymmetric the spectrum is not. This source of supersymmetry breaking can also be understood as follows. The BC's eqs. (2.15)–(2.17) are generally not stable under the supersymmetry transformations eq. (2.19); in other words the variation of the fields does not fulfill the BC's. As can easily be shown, the BC's are stable if and only if $n_f = c_f$. Nevertheless, as we have shown in section 2.2, the bulk plus boundary action *is* in fact supersymmetric: the boundary variation vanishes upon imposing the BC's obtained from the action principle, regardless of whether the BC's are actually stable under SUSY or not. This makes one speculate that this mechanism may have a spontaneous interpretation in supergravity. However a full supergravity analysis is clearly beyond the scope of the present paper.[7] In order to put our results on a more solid ground, we therefore will suggest an alternative description of the same physical theory in section 5 and 6 based on the more familiar orbifold (uptairs) approach. There all boundary conditions are fixed by the parities of the fields and are stable under supersymmetry. Of course these boundary conditions are imposed by hand and do not result from the action principle, making it hard to directly compare the two approaches.

In summary supersymmetry breaking arises from two different sources: one is the non-alignment of the vectors $\vec{t}_f$ (or equivalently $\omega \neq 0$); another one is the departure from zero of $n_f - c_f$. In both cases the supersymmetric limit is continuously connected which suggests that in the locally supersymmetric extension of the action local supersymmetry might be spontaneously broken. This point is extremely important and deserves a detailed investigation.

4.1 Boundary supersymmetry-breaking hyperscalar masses

In order to discriminate between the Scherk-Schwarz mechanism and other sources of supersymmetry breaking, we will first fix $\omega = 0$.

As mentioned earlier, the massless bosonic modes lie on a hyperbola in the (n_0, n_π) plane, eq. (3.6). It is clear that the mass squared of one eigenmode changes sign when one

[7] For a treatment of gravity and supergravity in the downstairs picture see for instance refs. [15].

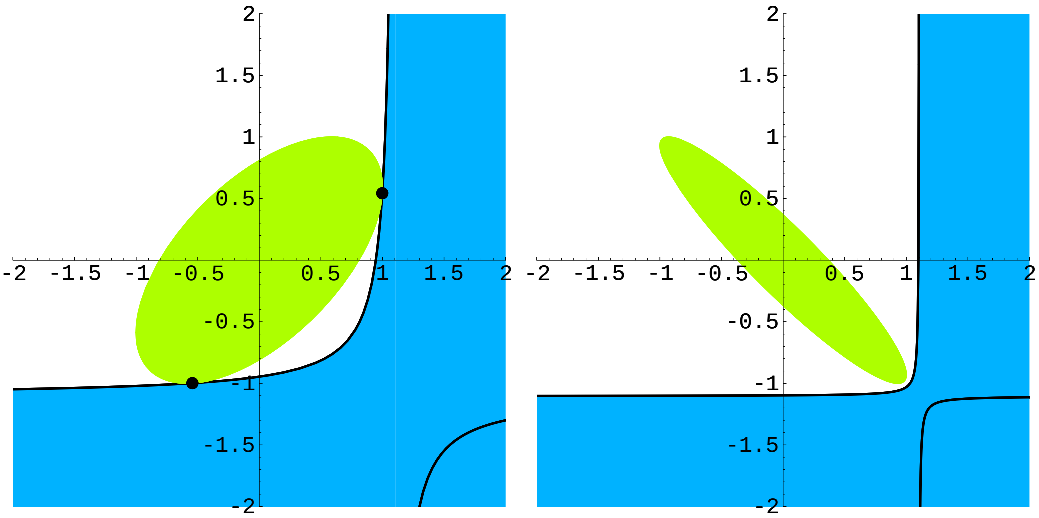

Figure 1: The hyperbola of massless modes for $\pi MR = 1.5$, $\omega = 0$ and $\tilde{\omega} = 0.15$ (left panel) and $\tilde{\omega} = 0.44$ (right panel) in the plane (n_0, n_π). The clear region to the upper left has no tachyonic modes, the darkly shaded (blue) region between to the two branches has one tachyonic eigenvalue, and the region to the lower right has two tachyonic modes. The ellipse corresponds to the allowed points in the plane (c_0, c_π). The two dots mark the points where the fermions are massless.

crosses this curve. As can be explicitely checked from eq. (3.5), at $n_0 = n_\pi = 0$ there are no tachyonic modes, while at $n_0 = n_\pi \gg 1$ there is one (complex) tachyon with $m^2 = -N_0^2$ and at $n_0 = -n_\pi \gg 1$ there are two degenerate tachyons with $m^2 = -N_0^2$. It is then easy to identify three distinct regions with different number of tachyonic eigenvalues.[8] For the case $\omega = 0$, we illustrate this situation in figure 1 by showing a plot in the (n_0, n_π) plane.

In the same plot we also provide the allowed values of $c_{0,\pi}$, defined by eq. (3.16). The interior of the ellipse corresponds to the allowed values in the plane (c_0, c_π). The supersymmetric points $n_f = c_f$ are thus also limited to the interior of the ellipse. Note that for $\omega = 0$ the ellipse cannot overlap with the shaded region, for this would mean the fermions to acquire tachyonic masses. However, for certain values of M and $\tilde{\omega}$, there are two points where the ellipse is tangent to the hyperbola, the intersection points corresponding to exactly massless supersymmetric spectra. These points are given by

$$\begin{pmatrix} c_0 \\ c_\pi \end{pmatrix} = \tau^{-1} \begin{pmatrix} \sin^2 \pi\tilde{\omega} \pm \cos \pi\tilde{\omega} \sqrt{\tau^2 - \sin^2 \pi\tilde{\omega}} \\ -\sin^2 \pi\tilde{\omega} \pm \cos \pi\tilde{\omega} \sqrt{\tau^2 - \sin^2 \pi\tilde{\omega}} \end{pmatrix} \tag{4.2}$$

and are obviously constrained to

$$\tau^2 \geq \sin^2 \pi\tilde{\omega} \,. \tag{4.3}$$

For other values of τ and $\tilde{\omega}$ there is no intersection of the ellipse with the hyperbola and hence there are no massless fermions. This is actually the case in the right panel of figure 1. Notice that there all points $n_f = c_f$ now correspond to supersymmetric but massive spectra.

[8] Some tachyonic spectra will be investigated in subsection 4.2 and used for electroweak symmetry breaking in section 7.

The case $\tilde{\omega} = 1/2$ is special. In fact if the hypermultiplet $\mathbb{H}$ transforms non-trivially under the gauge group the 4D theory might be anomalous if the fermion modes are not paired to get a Dirac mass. This happens for instance if the hypermultiplet scalar zero mode is identified with the Higgs field doublet H in the Standard Model.[9] A quick glance at eq. (3.15) shows that a sufficient condition for this to happen is $\tilde{\omega} = 1/2$, i.e. $\tilde{c} = -1$. In that case the ellipse degenerates to the line $c_0 = -c_\pi$ and the spectrum becomes vectorlike, as eq. (3.15) for fermions (Higgsinos) becomes a perfect square

$$\left\{1 - c_0 \frac{M}{\Omega} \tan(\Omega\pi)\right\}^2 = 0\,. \tag{4.4}$$

For instance, when $c_0 = 0$, then the fermionic spectrum is given by

$$\Omega_n = n + \frac{1}{2} \quad \Longrightarrow \quad m_n^2 = M^2 + \left(n + \frac{1}{2}\right)^2 . \tag{4.5}$$

For $c_0 = -c_\pi = -1$ the spectrum can be calculated in the large M limit and is given by

$$\Omega_n = n + \mathcal{O}(M^{-1}) \quad \Longrightarrow \quad m_n^2 = M^2 + n^2 + \mathcal{O}(M^{-1})\,, \qquad n \geq 1\,. \tag{4.6}$$

Finally, for the case $c_0 = -c_\pi = 1$ there is an exactly massless Dirac fermion in the limit $\tau = 1$ ($M \to \infty$). In fact, it can be shown that for $MR \gtrsim 1$ there is a light Dirac fermion with mass

$$m = 2M \exp(-\pi M)\,. \tag{4.7}$$

Interestingly enough the wavefunctions of the two chirality degrees of freedom localize towards different branes.

4.2 Scherk-Schwarz supersymmetry breaking

For $\omega \neq 0$ the situation changes. Even for $c_f = n_f$ the theory does provide different spectra for fermions and bosons as supersymmetry is now broken by the Scherk-Schwarz mechanism. In figure 2 we show the situation in the case $\omega = 1/2$. In the left panel ($\tilde{\omega} = 0.15$) one can see that for $n_f = c_f$ bosons always have strictly positive mass-squared, even at the points where the fermions are massless.[10] In order to have massless or tachyonic scalars, one has to move away from points $n_f = c_f$, thereby introducing the new source of supersymmetry breaking discussed above.

In the right panel of figure 2 we choose $\tilde{\omega} = 1/2$. As opposed to the previous case, it is now possible to have a tachyon at $n_f = c_f$. A particularly interesting choice is $n_0 = c_0 = -n_\pi = -c_\pi = 1$ where (for $MR \gtrsim 1$) there are two light scalars with masses squared

$$m^2 = \pm 4M^2 \exp(-\pi MR) \tag{4.8}$$

[9] This possibility will be analyzed in detail in section 7.

[10] Notice that the masslessness condition for fermions is still determined by the intersection of the ellipse with the $\omega = 0$ hyperbola, not shown in the plot.

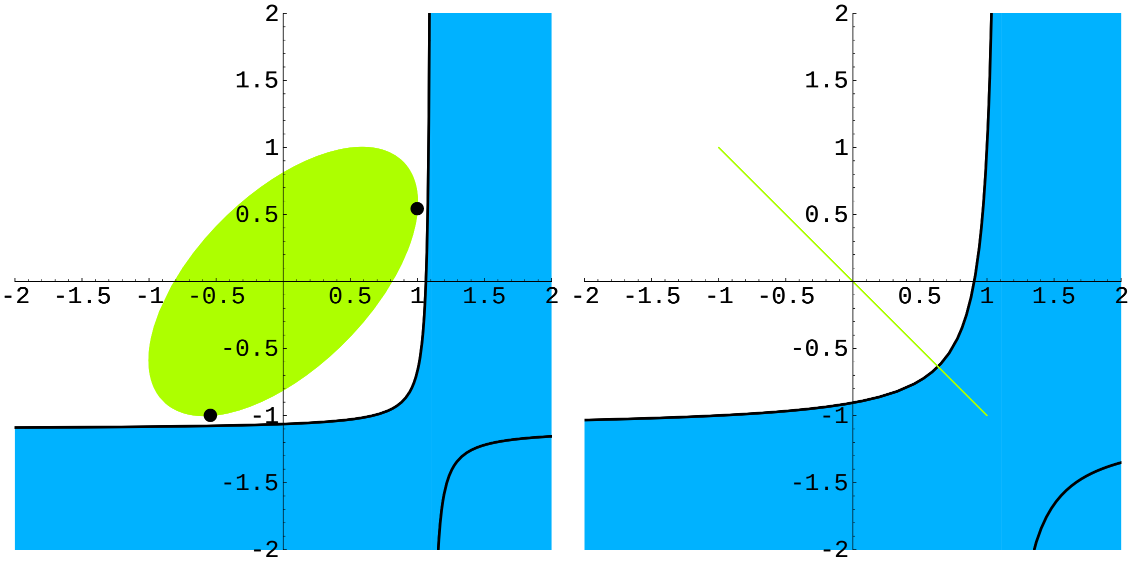

Figure 2: Same as figure 1, but with $\omega = 1/2$ and $\tilde{\omega} = 0.15$ (left panel) and $\tilde{\omega} = 1/2$ (right panel.

while the vectorlike fermion zero mode is still given by eq. (4.7). These particular cases were already considered in refs. [16, 17] in the context of the orbifold approach.[11] In figure 3 we show the numerical solution for these masses for general values of M. Notice that for $MR \gg 1$ both bosonic and fermionic masses are exponentially suppressed with the corresponding phenomenological troubles. However for values $MR \ll 1$ the exponential suppression disappears, although the fermion remains lighter than the Higgs boson for $MR \gtrsim 0.2$, and correspondingly the present experimental bounds on charginos can put a lower bound on the Higgs mass in this class of models. The fact that the zero mode boson is tachyonic and that supersymmetry is broken by Scherk-Schwarz boundary conditions provides a priori a very promising class of models of electroweak symmetry breaking.[12] In fact supersymmetry breaking is supersoft (finite) as it is due to global effects typical of the Scherk-Schwarz breaking, while electroweak symmetry is accomplished at the tree level and the electroweak breaking scale (Higgs mass) can be decoupled from the inverse radius of compactification, which can help to solve the little hierarchy problem. The wave function of the Higgs is exponentially localized to one of the interval boundaries, which can help in (partially) solving the problem of fermion masses while the radion can be stabilized by the Casimir energy at one or two-loop order as recently proposed [18]. A detailed analysis of these, and other questions (outside the scope of the present paper) will be considered elsewhere.

[11] The relation to the orbifold approach will be clarified in section 5.

[12] Of course radiative corrections have to be taken into account. For $M = 0$ this has been done in ref. [3] and it was found in ref. [17] that EW symmetry breaking can occur as long as the localizing mass term for the top hypermultiplet is not too big. Of course the situation changes when we slightly localize the Higgs field and allow for a non-vanishing value of the parameter M. In that case the tachyonic tree level mass becomes more and more important when M increases, reaching a maximum at around $MR \sim 0.5$, and in this region EW symmetry breaking can occur even with a fully localized top.

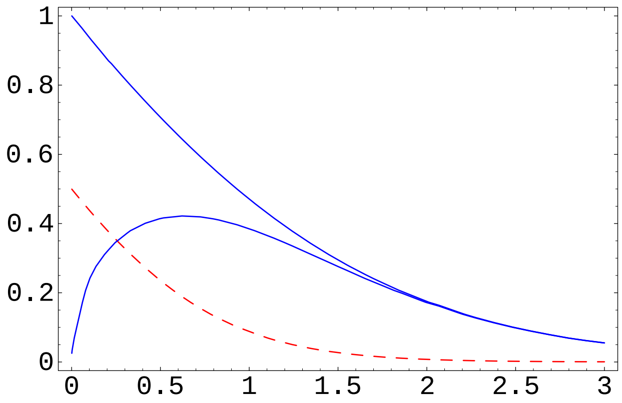

Figure 3: Light particle spectrum for $\omega = \tilde{\omega} = 1/2$, $n_0 = c_0 = 1$, $n_\pi = c_\pi = -1$. The plot shows $|mR|$ as a function of MR. The blue (solid) line corresponds to the bosons, the lower curve being the tachyon. The red (dashed) is the mass of the lightest fermion.

Another particularly interesting supersymmetry breaking case is $\omega = 1/2$ and $N_0 = N_\pi \equiv N$ where supersymmetry is broken by both the SS parameter and by the boundary terms N_f. The bosonic spectrum is now determined by the equation

$$(\Omega^2 + N^2)\, \frac{\tan^2(\Omega\pi)}{\Omega^2} = 0 \tag{4.9}$$

with solutions

$$\begin{aligned} m_n^2 &= n^2 + M^2 \;\; (n \neq 0) \\ m_0^2 &= M^2 - N^2 \,. \end{aligned} \tag{4.10}$$

The spectrum for the case $M = N = 0$ coincides with the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ model of ref. [4] while the case with $M = N \neq 0$ is a generalization but still the bosonic zero mode is massless. The case with $M \neq N$ is a different generalization where the bosonic zero mode is massive. An interesting possibility is when $M < N$ in which case the zero mode is a tachyonic state and can play the role of the Higgs doublet of the Standard Model and trigger electroweak (and supersymmetry) breaking at the tree level. In particular the case $M = 0$ reproduces the fermionic sector of the model of ref. [4] while for $N \neq 0$ the bosonic sector has a tachyonic zero mode.

The wave function for the bosonic modes is given, from (3.3), as

$$f_n(y) = \left(\cos(\Omega y)(1 - R_0) + \frac{\sin(\Omega y)}{\Omega} \left[1 + R_0 - N_0(1 - R_0)\right] \right) \varphi \,, \tag{4.11}$$

where φ has to be determined from the BC's at $y = \pi$. For the bosonic sector of the previous case where $R_0 = R_\pi$, $N_0 = N_\pi = N$, the wave function of the zero mode, satisfying $\Omega^2 + N^2 = 0$, is defined by

$$f_0(y) = e^{-Ny}\varphi \qquad \text{with} \qquad (1 + R)\varphi = 0 \,. \tag{4.12}$$

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Two components of φ are projected away by the last condition in (4.12) while the reality condition implies that only one independent component is kept. The latter one is fixed by the normalization condition. Eq. (4.12) shows that for $N \neq 0$ the bosonic zero mode is localized at the $y = 0$ ($y = \pi$) boundary for $N > 0$ ($N < 0$).

In this case we do not expect supersymmetry breaking to generically be supersoft because, on top of the non-vanishing Scherk-Schwarz parameter we have a departure from the supersymmetric relation (4.1). On the other hand, from the phenomenological point of view the Higgsino masses are much larger than the Higgs mass and present experimental bounds on chargino masses do not constrain at all the present model. In fact the nature of the supersymmetry breaking will be clarified in section 6 while some comments about electroweak breaking will be presented in section 7.

5. Comparison with the orbifold approach

In order to compare the previous formalism with the more usual orbifold approach, and to also shed light on the nature of the previously considered supersymmetry breaking, we show in this section that the same physical theory can be obtained if one considers the orbifold $S^1/\mathbb{Z}_2$. We assign the following parities to the fields

$$\Psi(-y) = i\gamma^5\sigma_3\Psi(y)\,, \qquad \bar\Psi(-y) = -\bar\Psi(y)i\gamma^5\sigma_3\,, \tag{5.1}$$

$$\Phi(-y) = \sigma_3\otimes\sigma_3\Phi(y)\,, \qquad \bar\Phi(-y) = \bar\Phi(y)\sigma_3\otimes\sigma_3\,, \tag{5.2}$$

$$F(-y) = -\sigma_3\otimes\sigma_3 F(y)\,, \qquad \bar F(-y) = -\bar F(y)\sigma_3\otimes\sigma_3\,. \tag{5.3}$$

We also could introduce Scherk-Schwarz twists for the $\mathrm{SU}(2)_R$ and $\mathrm{SU}(2)_H$ symmetries. However since the presence of an $\tilde\omega \neq 0$ parameter amounts to a supersymmetric mass, while the nature and interpretation of a Scherk-Schwarz twist $\omega \neq 0$ has been widely clarified in the literature [5, 6, 8], we will simplify our discussion in this section by assuming $\omega = \tilde\omega = 0$. Furthermore, we replace the action given in eqs. (2.4)–(2.6) by

$$\mathcal{S}^0_{\rm bk} = \int\left(-\frac{1}{2}\bar\Phi\,\partial^2\Phi + \frac{i}{2}\bar\Psi\gamma^M\partial_M\Psi + 2\bar F F\right)\,, \tag{5.4}$$

$$\mathcal{S}^{\rm m}_{\rm bk} = \int\left(2i\bar F\mathcal{M}\Phi + \frac{1}{2}\bar\Psi\mathcal{M}\Psi\right)\,, \tag{5.5}$$

$$\mathcal{S}_{\rm bd} = \int\Big(N_0\delta(y) - N_\pi\delta(y-\pi)\Big)\bar\Phi\Phi\,. \tag{5.6}$$

In order to have well-defined parity for the mass terms, we take the vector $\vec p$ defined in eq. (2.13) to be

$$p = (p_1, p_2, \epsilon(y)p_3)\,, \tag{5.7}$$

where $\epsilon(y)$ is the sign-function. Choosing $p_1 = p_2 = 0$ one reproduces the odd mass terms for hypermultiplets previously considered in the literature [19, 20, 16, 14]. The boundary mass terms involving the N_f parameters are similar to the ones encountered in eq. (2.6). In fact the boundary conditions (5.2) require $R = -\sigma_3\otimes\sigma_3$, so that by using this in eq. (2.6) we find eq. (5.6). The additional factor of 2 comes from the fact that the support of the delta function on the circle is twice the one on the interval, while the relative sign of the two boundaries reflects our convention of taking the orientation of the boundary at $y = 0$ to be negative. Boundary mass terms — which in the interval give rise to boundary conditions — on the orbifold generate jumps for the profiles of wave functions across the brane. It is easy to calculate these jumps for the special kind of mass terms of eq. (5.6). All fields are continuous except the ∂_5 derivatives of even bosonic fields, which satisfy

$$\begin{aligned}(1 + \sigma_3 \otimes \sigma_3)[\Phi'(0^+) + N_0\Phi(0)] &= 0\,,\\ (1 + \sigma_3 \otimes \sigma_3)[\Phi'(\pi^-) + N_\pi\Phi(\pi)] &= 0\,.\end{aligned} \tag{5.8}$$

Here we write the matrix $(1 + \sigma_3 \otimes \sigma_3)$ to project on the even fields only. The spectrum can now be directly inferred from subsections 3.1 and 3.2. The bosonic one is given by eq. (3.5) with $\omega = \tilde{\omega} = 0$. For the fermionic one, notice that in order to produce our orbifold boundary conditions, we have to choose $\vec{s}_0 = \vec{s}_\pi = (0, 0, -1)$ and hence must use $c_0 = c_\pi = -p_3$ in eq. (3.15).

Let us next study supersymmetry of this action. The supersymmetry variation of the bulk action is now given by

$$\delta\mathcal{S}^0_{\rm bk} = 0\,, \qquad \delta\mathcal{S}^m_{\rm bk} = -2\,p_3M\,[\delta(y) - \delta(y - \pi)]\,\bar{\epsilon}\bar{\Phi}\gamma^5\sigma_3\Psi\,, \tag{5.9}$$

while the boundary piece varies into

$$\delta\mathcal{S}_{\rm bd} = 2i\,[N_0\delta(y) - N_\pi\delta(y - \pi)]\,\bar{\epsilon}\bar{\Phi}\Psi\,. \tag{5.10}$$

Making use of our parity assignments eq. (5.1) we conclude that for these two pieces to cancel we must have

$$n_0 = n_\pi = -p_3\,. \tag{5.11}$$

To compare with the interval approach, we note again that there $c_0 = c_\pi = -p_3$ and thus we find that for the action to be supersymmetric, relation (4.1) must hold. Therefore departure from the supersymmetric relation (4.1) implies supersymmetry breaking. However in contrast to the interval case where the action itself is supersymmetric for any values of N_f, the breaking here is explict and can be viewed as coming from localized soft masses for the even hyperscalars. Splitting the masses N_f into a supersymmetric and a soft piece, $N_f = -p_3M + M_f$ we can write the localized soft breaking lagrangian as

$$\mathcal{S}^{\rm hyper}_{\rm soft} = \int\Big(M_0\delta(y) - M_\pi\delta(y - \pi)\Big)\bar{\Phi}\Phi\,. \tag{5.12}$$

Supersymmetry breaking produced by the soft mass terms for even scalars in the action (5.12) bears strong similarities with the usual Scherk-Schwarz supersymmetry breaking by twisted boundary conditions in the gaugino (and gravitino) sector. In fact twisted Scherk-Schwarz boundary conditions for the gauginos λ^i $(i = 1, 2)$ can be produced by localized gaugino soft masses with an action [5, 6, 8]

$$\mathcal{S}^{\rm gauge}_{\rm soft} = \int\bar{\lambda}\,(M_0\delta(y) - M_\pi\delta(y - \pi))\,\lambda + \text{h.c.} \tag{5.13}$$

However the nature of supersymmetry breaking by boundary scalar masses is very different from that of the Scherk-Schwarz supersymmetry breaking (which provides a supersoft or finite breaking) as we will see in the next section.

6. Supersymmetry breaking by boundary masses

In this section we will study the nature of supersymmetry breaking by localized scalar masses as in eq. (2.6) for

$$N_f = \vec{p} \cdot \vec{s}_f M + M_f \tag{6.1}$$

with $M_f \neq 0$. For simplicity we will assume the case of vector-like fermions analyzed in subsection 4.2, $\omega = \tilde{\omega} = 1/2$, ($R_0 = R_\pi = R$) when there are no supersymmetric masses, i.e. $M = 0$. We can think of the localized soft masses as being generated by the non-zero VEV for the F-component of some chiral $N = 1$ superfield S_f living at the boundary at $y = y_f$. Operators of the form

$$\int d^4\theta \, S_f^\dagger S_f H_f^\dagger H_f \delta(y - y_f) \tag{6.2}$$

then generate the required soft masses, i.e. $M_f \sim \langle |F_S|^2 \rangle$. Here H_f denotes the chiral superfield made out of the components which survive the $\mathbb{Z}_2$ projection at the boundary y_f. In the following we would like to examine in more detail the nature of this brane localized supersymmetry breaking in the Higgs sector. At the end of this section we will comment on the transmission of this kind of breaking to the matter sector.

The gauge interactions of the hypermultiplet Φ depend on the gauge group. For simplicity we will assume in this section a U(1) gauge group with generator $\mathcal{Q}$. Consistency with the reality condition (2.2) implies that the generator $\mathcal{Q}$ satisfies [13]

$$\sigma_2 \mathcal{Q} = -\mathcal{Q}^\star \sigma_2 \,, \tag{6.3}$$

and we will then make the choice

$$\mathcal{Q} = \frac{1}{2} \sigma_3 \,. \tag{6.4}$$

The quartic lagrangian comes from the integration of the U(1) auxiliary field $\vec{X}$ in

$$\mathcal{L}_D = 2 \vec{X}^{\,2} + g_5 \bar{\Phi} \vec{\sigma}_R \cdot \vec{X} \otimes \mathcal{Q} \Phi \,, \tag{6.5}$$

where g_5 is the 5D U(1) gauge coupling and $\vec{\sigma}_R$ are the SU$(2)_R$ generators. Integration of $\vec{X}$ yields

$$\mathcal{L}_D = -\frac{1}{8} g_5^2 (\bar{\Phi} \vec{\sigma}_R \otimes \mathcal{Q} \Phi)^2 \,. \tag{6.6}$$

We will now call the independent components of Φ as

$$\Phi_1^1 = H_1 \,, \qquad \Phi_2^1 = H_2 \tag{6.7}$$

and will use the reality conditions (2.2) for the other components,

$$\Phi_2^2 = \bar{H}_1, \quad \Phi_1^2 = -\bar{H}_2 \,. \tag{6.8}$$

The quartic potential is then given by

$$V_D = \frac{1}{8} g_5^2 \left(|H_1|^2 + |H_2|^2 \right)^2 \,. \tag{6.9}$$

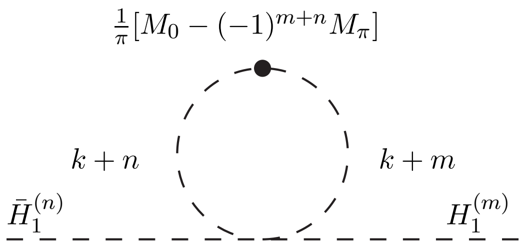

Figure 4: One-loop diagram renormalizing M_f.

Unlike in section 3 we will consider the term in eq. (2.6) as a perturbation and solve the equations of motion in the absence of it. The mass eigenvalues are then given as $\Omega_n = n$ and the mass eigenstates can be read off from eq. (3.2) by just putting $N_0 = N_\pi = 0$ there, i.e.

$$\begin{aligned} H_1 &= \frac{1}{\sqrt{2\pi}} H_1^{(0)}(x) + \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} \cos ny \; H_1^{(n)}(x) \\ H_2 &= \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} \sin ny \; H_2^{(n)}(x) \,. \end{aligned} \tag{6.10}$$

Now using the mode decomposition (6.10) we can write the boundary lagrangian (2.6) as

$$\mathcal{L}_{bd} = \frac{1}{\pi} \sum_{m,n=-\infty}^{\infty} \left[M_0 - (-1)^{m+n} M_\pi \right] \bar{H}_1^{(m)} H_1^{(n)} \,. \tag{6.11}$$

The renormalization of the boundary mass parameters M_f is given by loop diagrams induced by the quartic lagrangian (6.9) with one or more M_f-insertions.[13] Since the leading divergence is given by diagrams with one mass insertion, we will concentrate in diagrams as those in figure 4.

The contribution from the diagrams in figure 4 is proportional to the factor

$$I = \frac{1}{\pi} \left[M_0 - (-1)^{m+n} M_\pi \right] g^2 \mathcal{J} \,, \tag{6.12}$$

where $g = g_5/\sqrt{\pi}$ is the 4D gauge coupling, $\mathcal{J}$ is given by the Feynman integral

$$\begin{aligned} \mathcal{J} &= \sum_{k=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + (k+m)^2} \frac{1}{p^2 + (k+n)^2} \\ &= \sum_{\ell=-\infty}^{\infty} \int \frac{d^4p \, dz}{(2\pi)^4} \frac{1}{p^2 + (z+m)^2} \frac{1}{p^2 + (z+n)^2} e^{2i\pi\ell z} \end{aligned} \tag{6.13}$$

and we have made use of Poisson resummation.

[13] In this toy model the diagram with zero mass insertions will be quadratically divergent due to the generation of a localized Fayet-Iliopoulos (FI) term [21]. This can be seen as a renormalization of the supersymmetric mass term M and is clearly separable from the renormalization of the soft mass terms M_f. Of course one could avoid the generation of such terms by considering a second Higgs which does not interfere with the EW symmetry breaking process (as e.g. a second hypermultiplet with a (large) positive squared mass zero mode).

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The propagators in (6.13) have poles in the complex z-plane at locations $z = -n \pm ip$ and $z = -m \pm ip$. In this way for $\ell \neq 0$ the z-integrations contour can be closed by an infinite semicircle. Picking the residues of the corresponding poles provides the factor

$$e^{-2\pi|\ell|p}$$

that makes the integrand in the remaining integral to exponentially converge in the limit $p \to \infty$ and the corresponding integral to be finite. However for $\ell = 0$ there appears a linear divergence. In fact one can write

$$\begin{aligned}
\mathcal{J} &= \int \frac{d^4p\,dz}{(2\pi)^4} \frac{1}{p^2 + (z+m)^2} \frac{1}{p^2 + (z+n)^2} + \text{finite terms} \\
&= \frac{1}{64\pi} \Lambda + \text{finite terms}\,,
\end{aligned} \tag{6.14}$$

where Λ is the ultraviolet (UV) cutoff. One can interpret the result in (6.14) as a linear renormalization of the brane mass terms as

$$N_f = M_f\,(1 + \Delta), \quad \Delta = \frac{g^2}{64\pi} \Lambda R + \cdots\,. \tag{6.15}$$

Notice that to leading order the radiative corrections to the boundary mass terms Δ are boundary independent. Therefore the tree-level relation between M_0 and M_π is not spoiled by the (leading) correction in (6.15). Now that we have the loop-corrected localized soft masses one can go back to section 3 and compute the bosonic spectrum to all orders in the boundary masses as in subsection 3.1.

A comment concerning the UV sensitivity of the tachyonic (Higgs) mass will help to clarify the nature of the supersymmetry breaking induced by the boundary bosonic masses. This breaking is soft from the point of view that it does not induce any cubic counterterm in the 5D theory. However the mass term renormalizes linearly on the boundary, which induces in turn a linear renormalization in the Higgs mass. This renormalization can be simply understood from dimensional analysis since the operator $M_f\bar{\Phi}\Phi$, in terms of 4D fields, has dimension three while the gaugino mass operator in (5.13), $M_f\lambda\lambda$, has dimension four: while the former is linearly sensitive to the cutoff the latter is not. However this sensitivity does not destabilizes the Higgs mass for values of the cutoff $\Lambda R \lesssim 10^2$: in fact considering for simplicity the weak coupling, $g^2/64\pi \sim 2 \times 10^{-3}$ and $\Delta \lesssim 0.2$. Finally, in models with a single Higgs the quadratically divergent FI term is the dominant effect and we would require a lower cutoff ($\Lambda R \lesssim 10$) to keep this effect small. In fact the quadratically divergent FI term can be avoided if we introduce several (e.g. two) Higgs hypermultiplets with charges satisfying the condition $\mathrm{tr}\mathcal{Q} = 0$, while the linear divergence can be cancelled if the condition $\mathrm{tr}\mathcal{Q}M_f = 0$ is fulfilled for them.

Finally, let us examine the transmission of supersymmetry breaking to the matter sector through loop effects. The sensitivity on the UV cutoff crucially depends on where matter lives. If it is localized on a brane where supersymmetry breaking occurs we expect to generate local soft mass terms for squarks, and by simple power counting these must scale as $g_5^2\Lambda^2 M_f|\tilde{Q}|^2\delta(y - y_f)$. If matter lives in the bulk a similar dimensional analysis

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gives $g_5^2 \Lambda M_f |\tilde{Q}|^2 \delta(y - y_f)$, while if localization of matter and supersymmetry breaking occur at different branes, no local mass terms are generated and we instead obtain finite (nonlocal) soft masses which are insensitive to the cutoff. Again on dimensional grounds, these effective 4D soft terms scale as $g_5^2 R^{-2} M_f |\tilde{Q}|^2$. In the latter case, which we will examine in more detail in section 7, these are in fact subdominant to the finite contribution from the SS breaking, $g_5^2 R^{-3} |\tilde{Q}|^2$, as $M_f \ll R^{-1}$ in order to decouple the Higgs scale from the compactification radius.

We have embedded in this section, and for the sake of analyzing the supersymmetry breaking induced by boundary scalar masses, a U(1) gauge theory in the interval. We will consider in the next section how the whole gauge group $SU(2) \otimes U(1)$ can be similarly embedded.

7. Electroweak breaking

We will now consider the case where the hypermultiplet is a doublet under the $SU(2)_L$ gauge symmetry. To this end we must generalize the formalism of the previous sections, where only one hypermultiplet was considered, to one where there are two. The reality condition (2.2) is now written as

$$\bar{\Phi}^i_\alpha = \epsilon^{ij} \rho_{\alpha\beta} \Phi^\beta_j \,, \tag{7.1}$$

where the tensor $\rho_{\alpha\beta}$ can be written in the form [12]

$$\rho = \mathrm{diag}(\epsilon \oplus \epsilon) = \mathbb{1} \otimes \epsilon \quad \text{or} \quad \rho_{\alpha\beta} = \delta_{\alpha_1\beta_1} \epsilon_{\alpha_2\beta_2} \,. \tag{7.2}$$

In particular the reality condition for hyperscalars $\Phi^\alpha_i = \Phi^{\alpha_1,\alpha_2}_i$ is given by

$$\Phi_2^{\alpha_1,2} = (\Phi_1^{\alpha_1,1})^* \equiv \bar{\Phi}^1_{\alpha_1,\,1} \,, \qquad \Phi_2^{\alpha_1,1} = -(\Phi_1^{\alpha_1,2})^* \equiv -\bar{\Phi}^1_{\alpha_1,2} \,. \tag{7.3}$$

It is now easy to see that the generators of the symmetry group that preserve the reality constraint must satisfy

$$\rho T^A = -T^{A*} \rho \,. \tag{7.4}$$

The largest possible symmetry group is thus generated by

$$\{\sigma^2 \otimes \mathbb{1},\ \sigma^1 \otimes \sigma^i,\ \sigma^3 \otimes \sigma^i,\ \mathbb{1} \otimes \sigma^i\} \tag{7.5}$$

which is the spinor representation of SO(5). As we will see the BC's will however break this to a subgroup and so does a nonzero mass term in the bulk.

The reality constraints for the boundary matrices S and T are given by eqs. (2.7) and (2.9) with the substitution $\epsilon_{\alpha\beta} \to \rho_{\alpha\beta}$ where the operator ρ is defined in eq. (7.2). We thus find the generalizations

$$S^T \rho = -\rho S \qquad \mathcal{M}^T \rho = -\rho \mathcal{M} \tag{7.6}$$

while the constraint (2.12) remains unchanged. We conclude that S and $\mathcal{M}$ are $so(5)$ valued. We expect the biggest unbroken subgroup if we choose $S_0 \propto S_\pi \propto \mathcal{M}$. In fact all such choices are equivalent and lead to an $SU(2) \otimes U(1)$ subgroup. The most convenient one is to take $S_f \propto \mathbb{1} \otimes \sigma^3$ which leads to $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ generated by

$$\{\sigma^2 \otimes \mathbb{1},\ \sigma^1 \otimes \sigma^3,\ \sigma^3 \otimes \sigma^3,\ \mathbb{1} \otimes \sigma^3\}\,. \tag{7.7}$$

The formal proof of supersymmetry of the action as well as the solution to the EOM go along similar lines as those followed in previous sections. In particular the mode decomposition for bosons and fermions is that given in eqs. (3.1) and (3.9), respectively. In order to have a vectorlike fermion spectrum as well as unbroken $\mathrm{SU}(2) \otimes \mathrm{U}(1)$, we will fix $S_0 = -S_\pi = \mathbb{1} \otimes \sigma^3$, i.e. $\tilde{\omega} = 1/2$.

We will now consider (for illustrative purposes) the model where we break supersymmetry by choosing $\omega = 1/2$, $M = 0$ and $N_0 = 0$, $N_\pi = -N \neq 0$. This supersymmetry breaking is soft when acting on the $N = 1$ supersymmetric matter sector (quarks and leptons) localized on the $y = 0$ boundary as has been noticed in section 6. On the other hand a quick glance at eq. (3.5) shows that the Higgs zero mode is tachyonic in this case provided that $N > 0$. The Higgsinos are vector-like and the corresponding fermionic modes have Dirac mass given by $m_n = n + 1/2$ $(n \geq 0)$.

The mass eigenvalues of the Higgs boson doublet are given by $m_n = n$ $(n > 0)$ for the non-zero modes while there is a tachyonic zero-mode with mass squared $m_0^2 = -m^2 < 0$, where m is the solution of

$$m \tanh(m\pi) = N\,. \tag{7.8}$$

The eigenstate of the (tachyonic) zero mode of the Higgs doublet is

$$\Phi_1^{\alpha_1,1}(x,y) = \mathcal{N}^{-1}\cosh(my)\,H^{\alpha_1}(x)\,, \qquad \Phi_2^{\alpha_1,2}(x,y) = \mathcal{N}^{-1}\cosh(my)\,[H^{\alpha_1}(x)]^* \tag{7.9}$$

all other components vanishing. Here $H(x)$ is the 4D physical Higgs field and Φ fulfills the BC's with $S_0 = -S_\pi = \mathbf{1} \otimes \sigma^3$ and $T_0 = -T_\pi = -\sigma^3$,

$$\frac{1}{2}(1 + R_f)\Phi(x, y_f) = 0\,, \qquad R_f = -\sigma^3 \otimes \mathbb{1} \otimes \sigma^3\,. \tag{7.10}$$

The normalization factor is determined to be

$$\mathcal{N}^2 = \frac{\pi}{2}\left[1 + \frac{\sinh(2\pi m)}{2\pi m}\right].$$

Notice that $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ acts on the physical Higgs field H in the standard way, i.e. by the generators[14] $\{\frac{1}{2}\sigma^i, \frac{1}{2}\}$. The effective 4D theory is obtained by integrating over the extra dimension. The mass lagrangian becomes

$$\mathcal{L}_m = m^2\,|H|^2\,. \tag{7.11}$$

The quartic lagrangian comes from integrating out the $\mathrm{SU}(2)_L \otimes U(1)_Y$ auxiliary fields $\vec{X}^A$ where $A = 1,\,2,\,3$ labels the generators of $\mathrm{SU}(2)_L$, T^A, and $\vec{X}^4$ the generator of $\mathrm{U}(1)_Y$, Y. From the action of the super-Yang-Mills and hypermultiplets

$$\mathcal{L}_D = 2\vec{X}^A \cdot \vec{X}^A + g_A \bar{\Phi}^j_\alpha\,(\vec{\sigma}_R)^i_j\,\vec{X}^A\,\left(T^A\right)^\alpha_\beta\,\Phi^\beta_i\,, \tag{7.12}$$

[14] We normalize the generators to $\mathrm{tr}\{T^A T^B\} = \frac{1}{2}\delta^{AB}$.

where g_A is the 5D gauge coupling corresponding to the generator T^A. Integration of $\vec{X}^A$ in (7.12) yields

$$\mathcal{L}_D = -\frac{1}{8}\, g_A^2\, \left(\bar{\Phi}\, \vec{\sigma}_R \otimes T^A\, \Phi\right)^2 . \tag{7.13}$$

Next we particularize (7.13) to the zero mode Higgs doublet[15] of eq. (7.9). We get the lagrangian[16]

$$\mathcal{L}_D = -\frac{1}{8}\left(g_5^2 + g_5'^2\right)|H|^4\, \frac{\cosh^4(my)}{\mathcal{N}^4} . \tag{7.14}$$

Putting together eqs. (7.11) and (7.14), expanding the neutral component of the Higgs doublet as $H^0 = h/\sqrt{2} + i\chi^0$ (where h is the normalized Higgs field with a vacuum expectation value $\langle h\rangle = v = 246\,\mathrm{GeV}$) and integrating over the fifth dimension we obtain for the Higgs field the tree-level potential

$$V = -\frac{1}{2}m^2\, h^2 + \frac{1}{32}\left(g^2 + g'^2\right)\, \kappa(\pi m R)\, h^4 , \tag{7.15}$$

where g and g' are the corresponding 4D gauge couplings[17] and $\kappa(\pi m R)$, defined by

$$\kappa(x) = \frac{6x^2 + 4x\sinh(2x) + x\sinh(4x)/2}{[2x + \sinh(2x)]^2} , \tag{7.16}$$

comes from the normalization factor of the zero-mode wave function in (7.9).

Fixing the minimum of the potential to the physical value v one finds the tree-level Higgs mass as a function of the Z-boson mass m_Z

$$\begin{aligned} m_H^2 &= \kappa(\pi m R)\, m_Z^2 , \\ m^2 &= \frac{1}{2} m_H^2 . \end{aligned} \tag{7.17}$$

From the electroweak breaking condition (7.17) and the expansion $\kappa(x) = 1 + 4x^4/45 + \ldots$ we can see that $m \simeq m_H \simeq m_Z$ and for values of the compactification radius $1/R \sim$ few TeV $m\pi R \ll 1$. In this region the mass eigenvalue of eq. (7.8) can be solved analytically as

$$m^2 \simeq \frac{N}{\pi R} , \tag{7.18}$$

which, along with eq. (7.17) allows to fix the value of the mass parameter N required by the electroweak breaking condition to

$$N \simeq \frac{\pi}{2} m_Z^2 R . \tag{7.19}$$

Some comments are in order here. We have presented both the electroweak breaking condition and the Higgs mass at the tree level. On the one hand the electroweak breaking condition we have obtained (7.19) should be modified by the presence of one-loop corrections, in particular from those arising from the Yukawa couplings between the Higgs

[15]We can assume here that non-zero modes with masses controlled by $1/R \simeq$ few TeV are much larger than the weak scale and they have been integrated out.

[16]For the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ group with 5D gauge couplings g_5 and g_5'.

[17]4D and 5D gauge couplings g_4 and g_5 are related to each other as $g_5^2 = \pi R g_4^2$.

and matter sector localized on the boundaries. Since those corrections are model dependent and do depend on the precise localization and couplings of matter states we will not consider them in this paper but will postpone a more detailed analysis for further publication. In particular as it was pointed out in section 6 in any realistic model the condition $\mathrm{tr}Y = \mathrm{tr}YN_f = 0$ on the Higgs sector should be fulfilled to avoid the presence of any power sensitivity of the Higgs mass on the cutoff Λ. On the other hand the Higgs mass in (7.17) is the tree-level mass and its natural value is m_Z. As in the minimal supersymmetric standard model (MSSM), to obtain the prediction of the physical Higgs mass radiative corrections should be added: they are controlled by top-quark mass and (logarithmically) by the mass and mixing angle of the third generation squarks. One should meet in this model the large $\tan\beta$ MSSM prediction for the SM-like Higgs mass.

8. Conclusions

In this paper we have analyzed the formalism of hypermultiplets propagating in the five-dimensional interval $[0, \pi R]$. We have written down an explicit supersymmetric bulk + brane action where the field boundary conditions are dynamically obtained from the action principle. The orbifold boundary conditions are obtained as particular cases. The theory is characterized by three vectors in $\mathrm{SU}(2)_R$ space (two boundary unit vectors $\vec{s}_f$ and $\vec{t}_f$ and one bulk unit vector $\vec{p}$), and one boundary (n_f) and one bulk (M) scalars. A misalignment of the vectors $\vec{s}_f$ on the two boundaries gives rise to a supersymmetric mass for the hypermultiplet and that of the vectors $\vec{t}_f$ is interpreted as the Scherk-Schwarz supersymmetry breaking. Finally the presence of the boundary scalars n_f is also a potential source of supersymmetry breaking if there is a mismatch between n_f and $\vec{p}\cdot\vec{s}_f$. In fact we can define soft scalar masses M_f, as $n_f = \vec{p}\cdot\vec{s}_f + M_f/M$, that can break supersymmetry and electroweak symmetry at the tree level.

While the nature of the Scherk-Schwarz supersymmetry breaking was already clear, and known to be equivalent to boundary gaugino masses for 5D vector multiplets, that of supersymmetry breaking by boundary hyperscalar masses is clearly an issue. In fact while it is known that the Scherk-Schwarz supersymmetry breaking is one-loop finite, it provides a two-loop linear divergence corresponding to the one-loop renormalization of the gauge coupling [23]. We have proven in this paper that localized Higgs masses have one-loop linear divergences corresponding to the renormalization of a dimension-three operator on the 4D boundary. Furthermore this linear divergence (along with the quadratically divergent FI term) can be tuned to zero in models with two hypermultiplets with opposite hypercharges. Finally if the $N = 1$ matter sector and the soft Higgs masses are localized on different boundaries the transmission of supersymmetry breaking from the Higgs to the squark sector is finite and subleading with respect to the transmission of the Scherk-Schwarz breaking.

For the particular example we have worked out in some detail, where supersymmetry and electroweak breaking are triggered at the tree level, the natural tree-level value of the Higgs mass is m_Z. A more detailed analysis of the phenomenology of the models presented in this paper is left to future research. Let us finally stress that although the

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present analysis is done at the level of global supersymmetry and its continuation in local supersymmetry is clearly an issue that deserves further studies, the results obtained in the present paper can be used to construct new and interesting phenomenological models of electroweak breaking genuine to extra-dimensional scenarios. In particular we expect it can be used to construct phenomenologically realistic models of electroweak breaking in RS scenarios where SS is not an option to break supersymmetry [24].

A. Useful identities

In this appendix we present some useful identities to deal with fields obeying reality constraints. For fermions as in eq. (2.3), we find the following rules for bilinears:

$$\bar{\Psi}\, \sigma_H \gamma\, \Omega = -\alpha(\sigma_H) \bar{\Omega}\, \sigma_H \gamma\, \Psi\,, \tag{A.1}$$

where $\gamma = \{\mathbb{1}, \gamma^M\}$. and $\alpha(\sigma)$ is defined by $\alpha(\sigma_0) = +1$ and $\alpha(\sigma_i) = -1$. The reality properties are given by

$$(\bar{\Psi}\, \sigma_H \gamma\, \Omega)^* = -\alpha(\sigma_H) \bar{\Psi}\, \sigma_H \gamma\, \Omega\,. \tag{A.2}$$

For scalars as in eq. (2.2), the corresponding transposition rule is

$$\bar{\Phi}\, \sigma_R \otimes \sigma_H\, \Sigma = \alpha(\sigma_R)\alpha(\sigma_H)\, \bar{\Sigma}\, \sigma_R \otimes \sigma_H\, \Phi\,, \tag{A.3}$$

while the reality properties read

$$(\bar{\Phi}\, \sigma_R \otimes \sigma_H\, \Sigma)^* = \alpha(\sigma_R)\alpha(\sigma_H)\, \bar{\Phi}\, \sigma_R \otimes \sigma_H\, \Sigma\,. \tag{A.4}$$

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